

Resonance and Anomalous Dispersion of Water Waves

BY FRITZ BÜSCHING

Abstract:

Analyzing field measurements of *high energetic surf waves*, the author has come across an *anomalous dispersion effect* (ADE) that was previously unknown in connection with gravity waves.

For most kinds of waves, dispersion means the dependence of phase velocity c [m/s] on frequency f [Hz] or on wave length L respectively. With gravity waves, dispersion is *normal*, which means that $dc/df \leq 0$ or $dc/dL \geq 0$; - similar to what is known about electromagnetic waves (EM-waves) in the limited frequency range of the visible light (as demonstrated by the known sequence of spectral colors).

With EM-waves, however, *resonances* appear together with the phenomenon of an ADE, marked by $dc/df > 0$ or $dc/dL < 0$ respectively.

Seeking analogue conditions for water waves, the author had found two different model conceptions to be appropriate for (partial) standing waves in connection with basin oscillations.

In natural field conditions: *incident waves from the sea* (stimulator) resonating with *partial standing half-waves* in a definable water basin (resonator), and

In a scale model: the *wave maker* (stimulator) resonating with *partial standing quarter-waves* in the wave tank (resonator).

Zusammenfassung:

Bei der Auswertung seiner vor Sylt durchgeführten Naturmessungen *hochenergetischer* Brandungswellen war der Autor auf das bis dahin für Schwerewellen unbekannte Phänomen der *anormalen Dispersion* gestoßen. Unter Dispersion wird bei den meisten Wellenarten insbesondere die Abhängigkeit der Phasengeschwindigkeit (Wellenfortschritts­geschwindigkeit) c [m/s] von der Frequenz f [Hz] bzw. von der Wellenlänge L [m] verstanden. Bei Schwerewellen ist diese *normal* und durch $dc/df \leq 0$ bzw. $dc/dL \geq 0$ gekennzeichnet, etwa vergleichbar den elektromagnetischen Wellen (EM-Wellen) im Bereich des *sichtbaren* Lichts mit der bekannten Abfolge der Spektralfarben. *Resonanzen* treten aber insbesondere auch bei EM-Wellen zusammen mit dem Phänomen der *anormalen Dispersion* mit $dc/df > 0$ bzw. $dc/dL < 0$ auf. Als Ergebnis der Suche nach analogen Bedingungen bei Wasserwellen hat der Autor insbesondere zwei unterschiedliche Modellvorstellungen für (partiell) stehende Wellen im Zusammenhang mit resonanten *Beckenschwingungen* als anwendbar erkannt:

In der Natur als Resonanz der *von See kommenden Wellen* (Erreger) mit *partiell stehenden Halbwellen* in einem abgrenzbaren *Beckenvolumen* (Resonator) und

im verkleinerten Modell als Resonanz des *Wellenerzeugers* (Erreger) mit *partiell stehenden Viertelwellen* im Wellenkanal (Resonator).

The term '*anomalous dispersion*' had not been used in connection with water waves until the author in 1978 examined more closely his spectral evaluations of surf wave measurements saved on magnetic tape at Westerland/Sylt in 1973, [1], [2], [3].

In particular, the measurements concerned *water level deflections*, taken synchronously at two stations 15m away from each other in a coast perpendicular measuring profile. At that time, such measurements had been one of the first to use inductive pressure sensors.

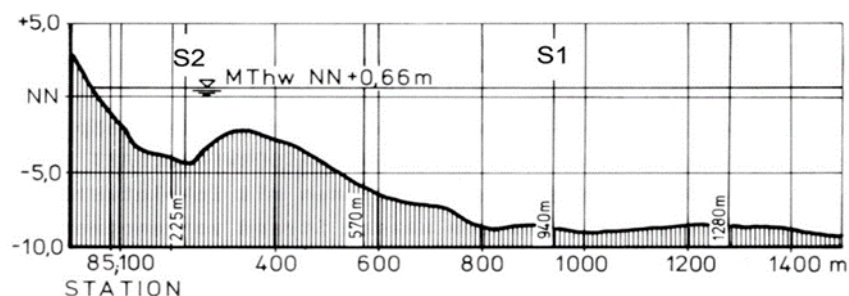


Fig.1: Measuring profile Westerland/Sylt, 1973. Stations on the beach 100m and 85m distant from the foot of the dune.

Evaluation methods of such kinds of measurements, normally then applied on strip chart records, seemed not to be very trustworthy. Hence, the author for the first time used automatic techniques for documenting the deformation of breaking storm waves.

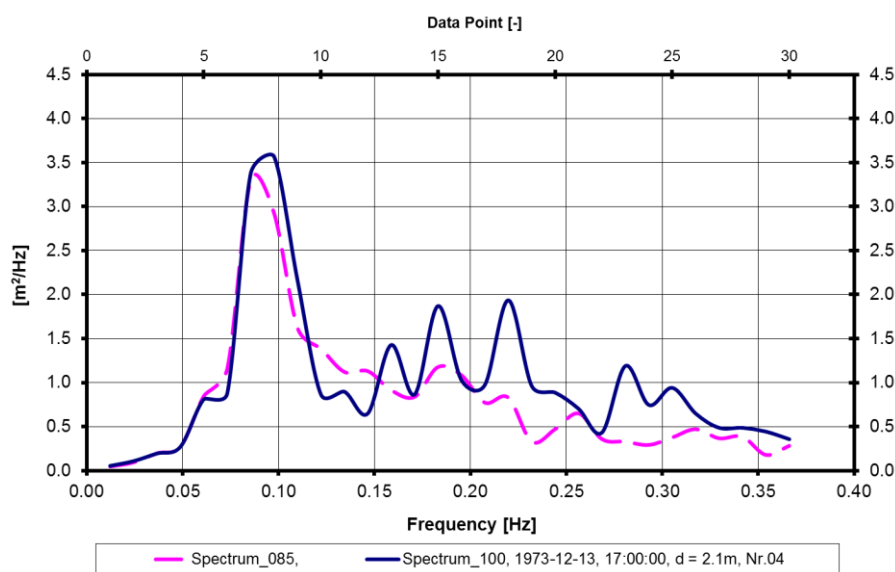


Fig.2: Energy density spectra of storm surf wave measurement No 4 at stations 100m and 85m within the measuring profile of Fig.1.
Peak frequency $f_p \approx 0.09$ Hz.

Especially a Hewlett-Packard Fourier Analyzer had been used to calculate the *spectral transfer function* based on the energy density spectra determined from the water level deflections at the two measuring stations synchronously [1], [4], [5]. That complex function describes the functional interrelation between the two signals according to magnitude and phase. Hence, the author was able to calculate *phase velocity spectra* using the transfer function's phase information and the distance of the measuring stations.

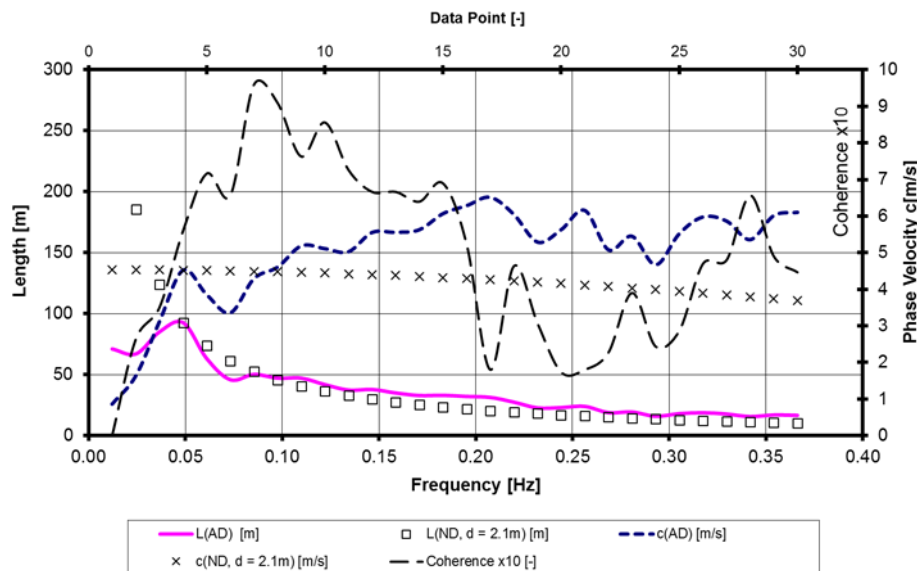


Fig.3: Spectra of quasi measured phase velocities $c(AD)(f)^1$, lengths of wave components $L(AD)(f)$ and Coherence $\gamma_{xy}^{-2}(f)$, to be compared with the theoretical functions $c(ND)(f)$ and $L(ND)(f)$ according to a water depths $d = 2.1\text{m}$.

As a substantial result from the analysis of 16 measuring intervals, distributed over 30 hours, the author had found phase velocity spectra, characterized by $dc/df > 0$ in the frequency range of appreciable energy densities. Actually, this is an identifying mark of *anomalous dispersion*, and it contradicts the classical dispersion relation, cf. Fig.2 and Fig.3.

After investigating the effect of anomalous dispersion with respect to the *transformation* of breaking waves [3], he focused attention on the root causes of anomalous dispersion.

Thus, in 1980 he recognized first a possible cause of anomalous dispersion applying his own formula, oriented to the Doppler – terminology of *accelerated* carrier media [6], [7]. Although accelerated currents apparently exist in the coastal zone, - especially at storm conditions, at an early stage he had also assumed an analogous behavior of *resonance absorption*. In fact anomalous dispersion is essential for resonance phenomena [8].

The term *resonance absorption* especially is used for the absorption of electromagnetic and particle radiation by microphysical systems, in which resonance occurs. In connection with water waves the

¹ The expression in brackets (AD) had been used in earlier publications and denotes values to be retraced to measurements (quasi measurements = derived from a different measured quantity) basically characterizing anomalous dispersion ($dc/df > 0$ and $dc/dL < 0$ respectively). Contrary the parenthesized expression (ND) stands for normal dispersion ($dc/df \leq 0$ and $dc/dL \geq 0$ respectively) according to the classical dispersion relation.

author believed to have recognized a similar process of resonance between partial standing and breaking waves on the one hand and the washing movement on a rather steep slope on the other [9], [10]. Thus, the waves coming from sea are looked upon as a stimulator and the washing movement as a resonator, in the knowledge that both also can be considered the components of a coupled oscillation exchanging energy. In order to influence such a system (perhaps to be named “surf resonance”²) in the sense of lower surf energy (i.e. lower heights of breaking waves), the author in 1989 had applied for a patent on *permeable revetments* (Hollow Cubes [11], [12]).

Its objectives consist in modifying the interaction between the partial standing waves at the structure and the washing movement on the structure, in particular changing the phase difference between them.

Since in the meantime other parties had not carried out investigations on equally intense surf conditions, Büsching and Speranski believed it useful to carry out error analysis, which also included the work of other researchers so far [13], [14].

As a result, the reliability of the anomalous dispersion effect (ADE) could be related to two frequency ranges, separated by the spectral peak frequency f_p .

Especially the ADE at $f > f_p$ was recognized as a *second order effect*, whereas at $f < f_p$ the ADE seemed to be accompanied by *partial standing waves*.

In connection with the testing of hollow revetment structures subjected to irregular waves in the wave tank of Bielefeld University of Applied Sciences (BUAS), however, also a different kind of resonant conditions was taken into account. Especially the view on the total system, consisting of the wave maker and the water body in the tank, led to considering *basin oscillations* also to be applicable with respect to coastal under water topography. Actually, the phenomenon of anomalous dispersion had been found in connection with the existence of partial standing waves in the wave tank [15].

It is well known that long wave oscillations (*Seiches*) may be set up in enclosed water bodies, but harbor resonances do not necessarily need regular and vertical boundary conditions of closed or semi-closed basins to be maintained. Accordingly, for instance Bascom [16] reported that even water masses on the continental shelf could perform seiches-like oscillations although no boundaries of a basin can be identified.

Little had been published, however, on resonance phenomena in connection with normal gravity waves [17]. But the author found it plausible to regard the channel existing between beach and sand ridge in front of Westerland/Sylt as a basin formation, cf. Fig.1; moreover in this sense not forgetting the *smaller* basin formations parallel to the coastline [18] [19].

² In connection with the definition of the complex reflection coefficient (CRC) [25] [21], the author had considered the theoretical limiting case of (positive) total reflection at the vertical wall principally equivalent to the (theoretical) negative total reflection at an inclined wall. Hence, the real cases of imperfect reflection in nature take place in between the above-mentioned two theoretical limits and are characterized not only by the wave height ratio H_r/H_i of the reflected and the incident wave but also by a phase shift $\Delta\phi$ between them. Most likely one could assign „surf resonance“ to a case of negative reflection (with phase shifts $90^\circ \leq \Delta\phi \leq 270^\circ$), possibly characterized by surging waves. Moreover, *dissipation* processes and *transmission* complete the surf condition on slopes, the latter manifesting itself as a momentum of wave run-up. Hence, the following wave run-down possibly can resonate with partial standing waves and by this reason be responsible for bigger breaker heights to be formed.

With the aim of exposing the revetment structures in the BUAS wave tank to energetic wave action as high as possible, in this case no precautions had been made to suppress re-reflection from the wave maker. In order to analyze the resonant basin oscillations, generated this way, the tests had been carried out comprising a rather big number of 91 wave probe stations, positioned in front of the slope, from station 0.79m to 9.79m, equally spaced 10cm. Their analog signals, taken quasi synchronously, had been processed by spectrum analyses, confined to a total frequency range $0.03263 \leq f \leq 1.3997\text{Hz}$. Due to the fact that the area, included in each of the energy spectra (integrated spectrum area, IA), is proportional to the potential energy at any measuring station, such IA-values were used in order to describe the distribution of the energy along the wave flume with reference to different frequency ranges, cf. Fig.4.

Especially it was possible to show the energy development of any single frequency component along the total length of the wave tank and thus measuring its wavelength, see Fig.5.

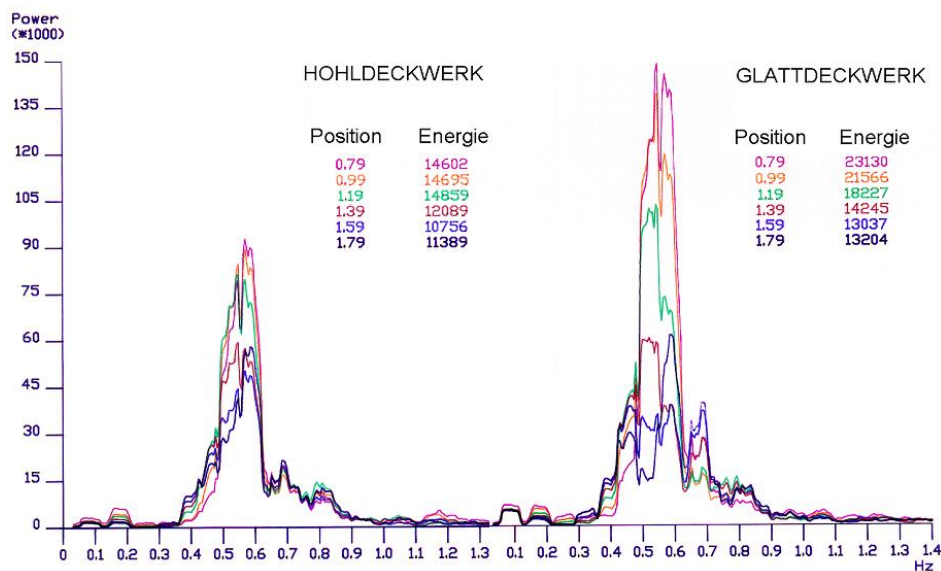


Fig.4: Synchronously measured energy spectra at stations 0.79m to 1.79m distant from IP. In front of a Hollow Revetment (left) and a Plane Revetment Structure (right) at Slopes 1:3.

As a result, it was found that the wave groups generated in the wave tank were in resonance with a number of natural oscillations of the water volume in the tank. This had been concluded from the fact that there were limited frequency ranges, in which neighboring frequency components possess approximately equal wave lengths and thus cause anomalous dispersion $dc/df > 0$ according to the fundamental equation of wave celerity $c = L \cdot f$, cf. Fig.5.

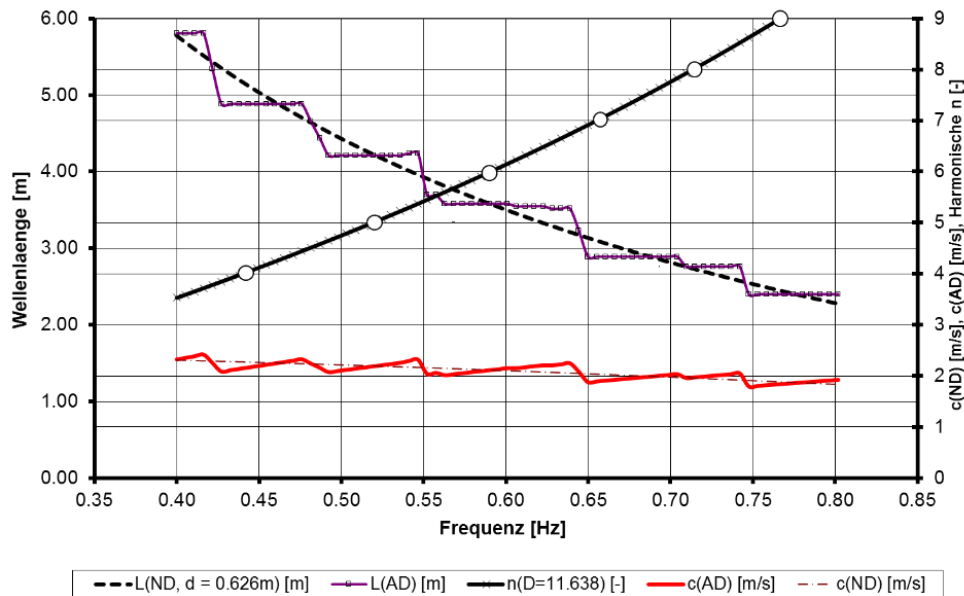


Fig.05: Component length L, phase velocities c and harmonic numbers n plotted with frequency.

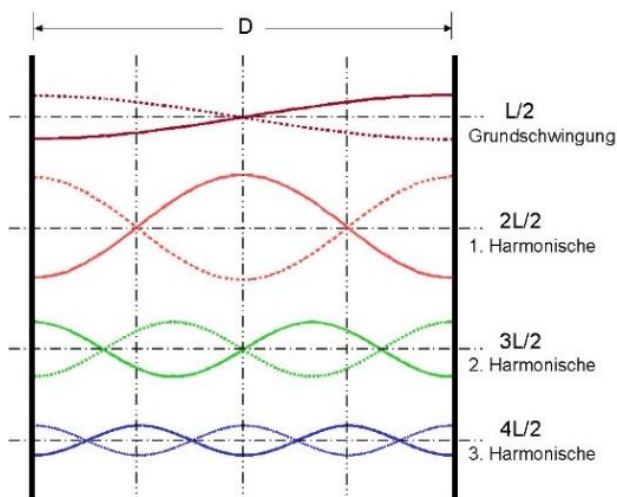


Fig.06: The first four theoretical mode shapes of natural oscillations in a basin confined by vertical walls at the front end and at the rear end.

Actually, the author was able to show that for the generation of resonant conditions of various volumes of water in a first approximation it is sufficient to have a prismatic volume of water in a basin bounded by vertical walls, (cf. Fig.06) acting as a resonator. Such an approximation may be applied on the content of a wave flume or on the volume contained in the measuring profile of Fig. 01),

It is known that in this case the natural frequencies belonging to the mode shapes of Fig.06 (perfect standing waves) can be calculated using Merian's (1828) formula (01)

$$f[Hz] = (n+1) \cdot \frac{c}{2 \cdot D} \quad (1)$$

Where

D = horizontal wall distance

c = wave celerity

n = harmonic number.

n = zero denotes the fundamental oscillation and n = 1, 2, 3...are named first, second, third harmonic etc.

Contrary to Merian, who simply used $c = \sqrt{gd}$ in his formula, at a spectral treatment it is necessary to use the quasi measured phase velocities (depending on frequency) $c(AD)(f)$, - especially for the purpose of extracting the harmonic numbers $n(f)$.

Based on model experiments of scale 1:5, the author had claimed that besides Positive Total Reflection (PTR) with water waves there also exists the theoretical limiting case of Negative Total Reflection (NTR) [20]. Hence, the circumstances for its approximate occurrence in nature can be described by defining a complex reflection coefficient (CRC) $\Gamma = C_r e^{i\Delta\varphi}$ [21].

More specifically: The investigations in a wave tank had shown that close to rather steep slopes an *incomplete node* (in accordance with a minimum of energy) exists more distinctly than an *incomplete loop* (in accordance with a maximum of energy). Thus evidently a *phase jump* occurs.

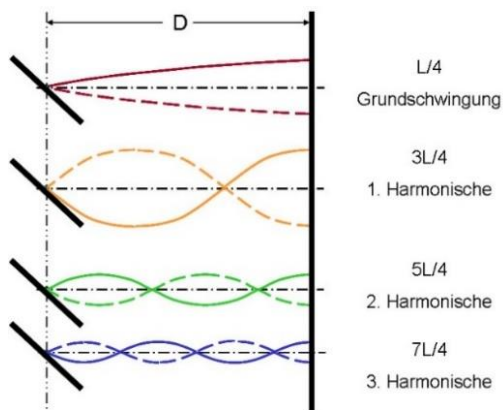


Fig.7: The first four theoretical mode shapes of natural oscillations of a basin, confined by a vertical wall at the front end and by a slope at the distance D.

Accordingly the natural frequencies of a wave tank, approximated by a basin confined by a vertical wall (at the hinge of the wave maker) and an inclined wall (slope) (at the opposite end), can be estimated more appropriately in using the following formula than by formula (1).

$$f[Hz] = (2n + 1) \cdot \frac{c}{4 \cdot D} \quad (2)$$

In formula (2) the standing wave mode shapes of Fig.07 are taken into account.

Solving it with respect to harmonic numbers $n(f)[-]$, yields formula (3):

$$n(f)[-] = \frac{2 \cdot D \cdot f}{c} - 0,5 \quad (3)$$

As an example concerning this matter, the improved results of basin oscillations of the wave tank used are shown in Fig.05.

In that regard, it should be pointed out that resonant higher harmonics (partial standing waves) occurred with ordinal numbers $4 \leq n \leq 9$, which had been found in using formula (3) applying the horizontal distance $D = 11.638\text{m}$. D is the horizontal distance between the hinge of the wave maker and the point IP, where the slope intersects the still water level.

Close to the point of resonance the measured component-lengths $L(\text{AD})(f)$ tend to a fairly constant value, which is obviously the reason for the phase velocities $c(\text{AD})(f) = L \cdot f$ to behave anomalously ($dc/df > 0$). As a consequence $L(\text{AD})$ and $c(\text{AD})$ progressively differ from the theoretical values $L(\text{ND})$ and $c(\text{ND})$; *downwards* with the frequency *decreasing* and *upwards* with the frequency *increasing*.

It should be noted here that the calculated resonance-frequencies match fairly *exactly* the intersection points of $L(\text{ND})$ and $L(\text{AD})$ on the one hand, and of $c(\text{ND})$ and $c(\text{AD})$ on the other – only, however, for waves of lengths 3.58m ($n = 5$; $f \approx 0.52\text{Hz}$) and 4.21m ($n = 6$; $f \approx 0.59\text{Hz}$), which carry maximum wave energies. That feature is well known also from resonance absorption of electromagnetic waves in dielectrics. Among other things, the increasing deviation for the higher and lower resonant frequencies may be due to the fact that the spectral analysis is not sufficiently reliable at the remaining harmonic resonance frequencies in the above mentioned sense.

Actually, comparable conditions of resonances and anomalous dispersion could also be found with respect to the field investigations on Sylt Island. Thus, also in this case close to a point of resonance, the measured anomalous phase velocities progressively differ from the theoretical values with *decreasing* frequency *downwards* and with *increasing* frequency *upwards*, while the component lengths tend to a more constant value, cf. Fig.2 and Fig.3.

Worth mentioning is, however, the fact that *different model conceptions* are appropriate for describing resonance conditions (partial standing waves) in connection with basin oscillations in a (small scale) wave tank and in natural topography.

Thus, the matching of ordinal number and resonance frequency in a *scale model* is very clear, in case the boundary conditions, underlying formula (2), are used; cf. Fig.5. But a fairly comparable degree of matching will be achieved for the *field investigations* only in case formula (1) is applied (cf. Fig.8) and it is rather clear for the (averaged) wave lengths $L(n = 0) = 92.62\text{m}$ ($f \approx 0.045\text{Hz}$), $L(n = 1) = 47.60\text{m}$ ($f \approx 0.09\text{Hz}$), and $L(n = 2) = 32.73\text{m}$ ($f \approx 0.18\text{Hz}$). At frequencies $f > 0.18\text{Hz}$, the wave lengths $L(n = 3) \approx 23\text{m}$ and $L(n = 4) \approx 18\text{m}$, also to be taken from Fig.8, furthermore comply with $L(n) \approx 92.62/(n+1)$, but reliability can no longer be preserved, because Coherence is poor at those frequencies.

With respect to formula (1), however, the author's measurements regarding the model experiments on Complex Reflection Coefficients should be taken into account [22], [20]. According to the *extrapolation* of some results to *longer* waves ($L > 12\text{m}$), at flat sloped embankments there rather exists positive reflection³

Hence, the storm wave resonance at Sylt Island concerning the basin between beach and ridge (Fig.01), formerly had reasonably been explained by the adapting lengths of neighboring frequency

³ By definition of the Complex Reflection Coefficient (CRC) $\Gamma = C_r e^{i\Delta\varphi}$ [25] [21] (magnitude $C_r = H_r/H_i$ and phase $\Delta\varphi$), *positive* reflection is marked by the condition that an incomplete *loop* of a partial reflected wave is closer to the sloping structure (IP) than the closest incomplete *node*, thus at phase differences $-90^\circ < \Delta\varphi < +90^\circ$.

components to the existing boundaries, in using mode numbers basing on formula (1), although the distance dune – ridge at first sight cannot be related to the found wavelengths.

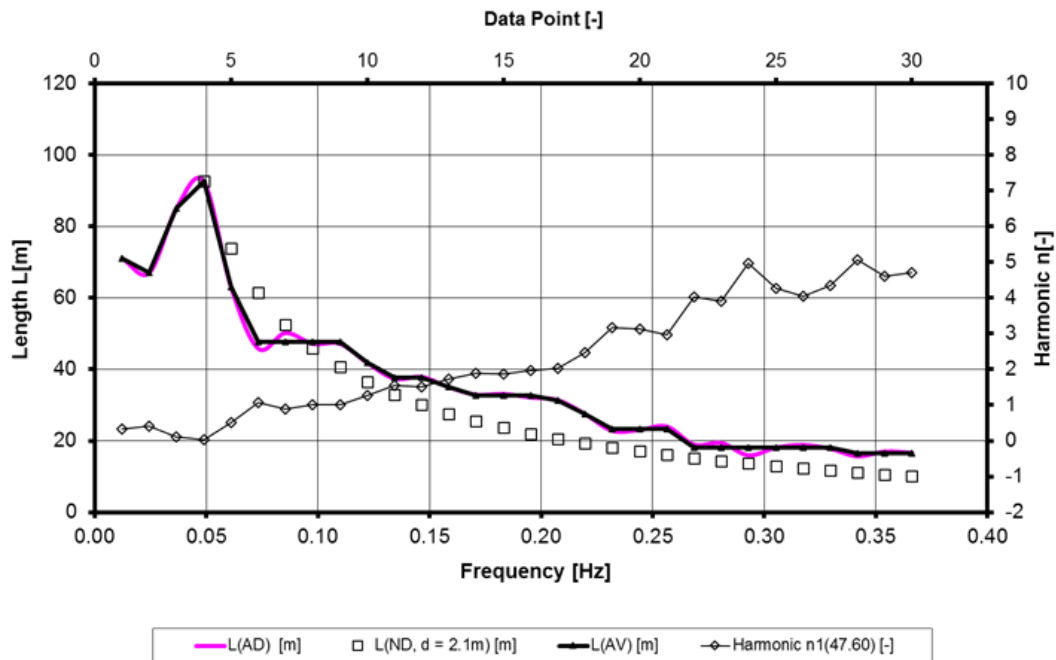


Fig.8: Magnified spectra of the lengths $L(f)$ and of the ordinal numbers $n(f)$ of basin oscillations.

Deduced from the big difference of the gradients $dc(AD)/df \gg 0$ and $dc(ND)/df < 0$ centered around the peak frequency $f_p \approx 0.09\text{Hz}$ in Fig.3, the corresponding length in the spectrum of component lengths in Fig.8 on an average is $L(f) = 47.6\text{m}$. Thus, a partial standing wave with this length would correspond to the first harmonic.

For the purpose of further clarification in Fig.09 the line spectra of energy densities $ED(L)$ are plotted with reference to stations 100m and 85m. Such values, related to discrete wave lengths had been calculated by summing up the single amounts of energy densities of frequency components having nearly equal wavelengths. With reference to station 100m the maximum energy density $\max ED(L) \approx 10\text{m}^2/\text{Hz}$ belongs to a wave length $L(AD) = 47.6\text{m}$ ($\approx 50\text{m}$). By contrast in the *frequency spectrum* there is a maximum energy density of about $\max EP(f) \approx 3.6\text{m}^2/\text{Hz}$ only (Fig.2), but the wave length (resonance wave length) $L_p(ND) \approx 50\text{m}$ is the same in both cases calculated for $d = 2.1\text{m}$. Furthermore, Fig.09 additionally contains the phase velocity spectra of Fig.03 and prominently in the spectrum $c(AD)(L)$ the resonance is marked by a jump $dc/dL \rightarrow \pm \infty$

Documented on December 14, 1973 by measurement 9, 10 and 11 at the period of highest tidal water levels, the extension of *averaged anomalous dispersion* to a larger frequency range can be traced back to the presence of at least *two dominant wave systems* possessing different higher harmonics.

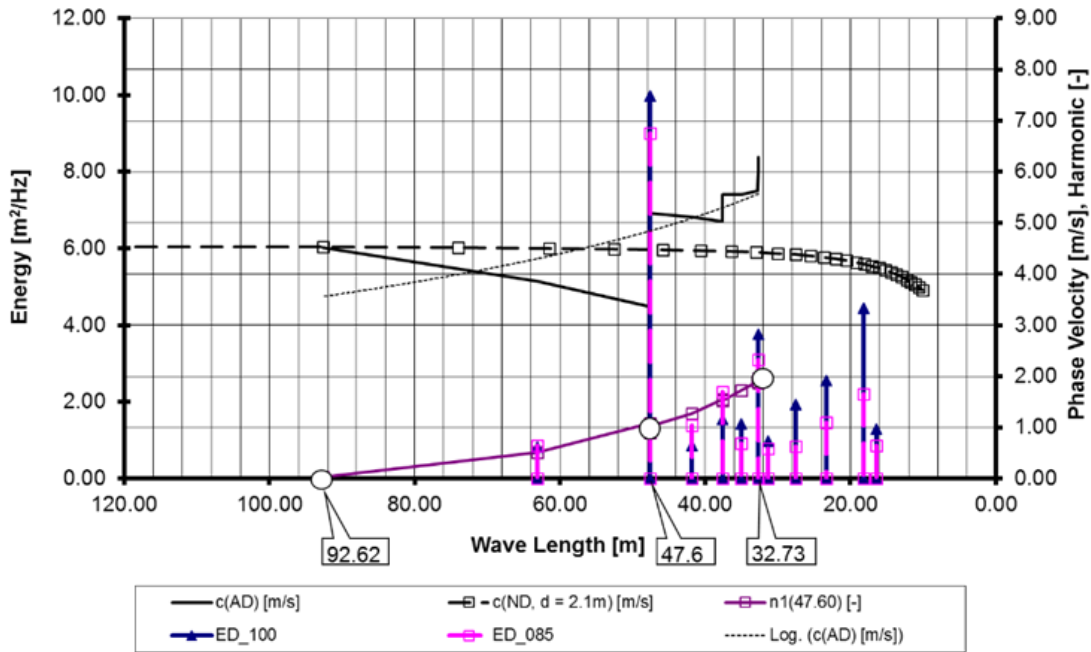


Fig.9: Phase velocities $c(ND)(L)$ and $c(AD)(L)$, Ordinal numbers of basin oscillations $n(L)$ and Line spectra of energy density $ED(L)$.

From the measurements no. 9 (starting 0:48) and 11 (starting 3:46), characterized in [18] by *bimodal energy spectra*, spots of resonance had been derived from the shape of the functions $c(AD)(f)$ corresponding to harmonics numbers 0 to 3 for each of the two wave systems.

In following Figures 10 to 12 of the present publication, however, this feature is further examined with respect to measurement *no. 10* carried out at maximum water depth ($d \approx 2.7\text{m}$) measured at all. Moreover, this measurement is characterized by a *broad unimodal* energy spectrum with maximum energy densities.

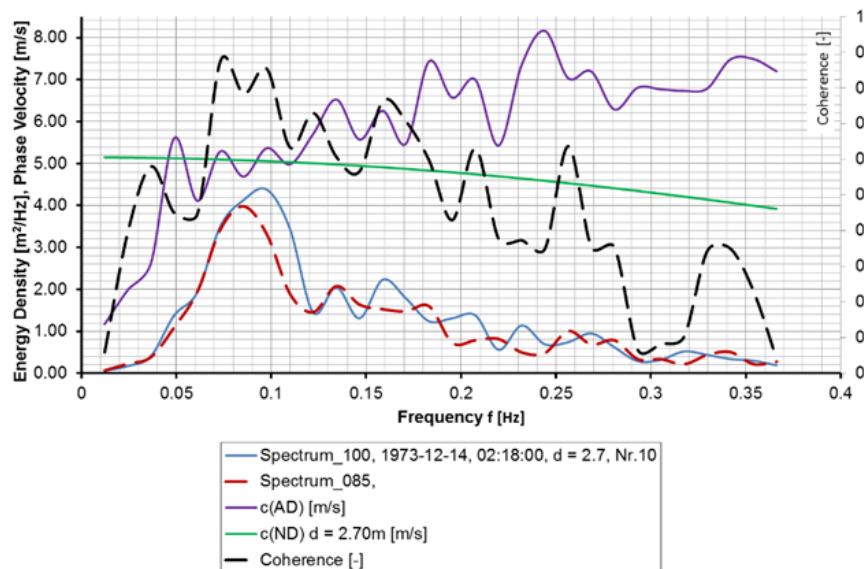


Fig. 10: Energy Spectra of measurement No. 10. Quasi measured oscillatory phase velocity function $c(AD)$ at considerable Coherence values, to be compared with $c(ND)$ according to the classical dispersion relation.

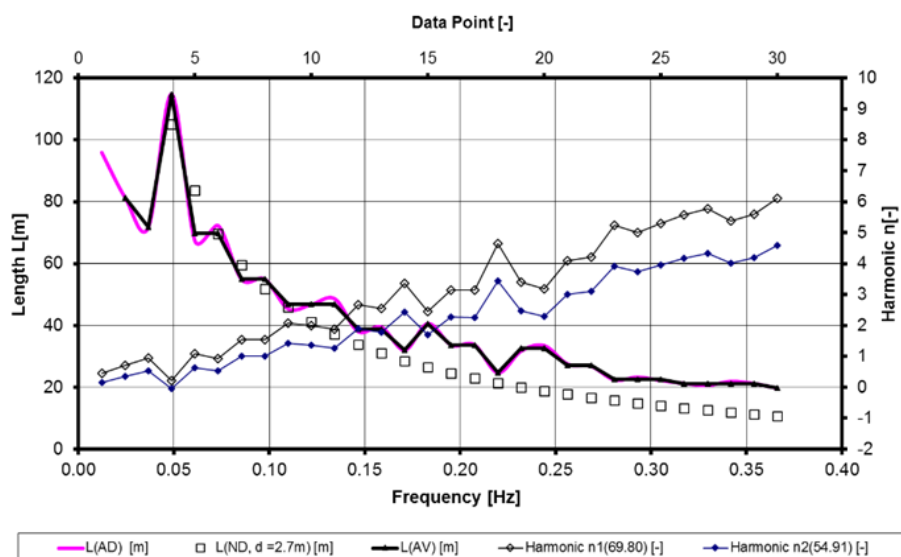


Fig. 11: Component length spectra $L(f)$ together with ordinal harmonic numbers $n(f)$. $L(AV)(f)$ denote the smoothed values of $L(AD)(f)$ and $L(ND)(f)$ are the theoretical values according to water depth $d = 2.7\text{m}$. The two wave systems are characterized by different curves of ordinal harmonic numbers; each calculated in using the respective length of the first harmonics given in brackets.

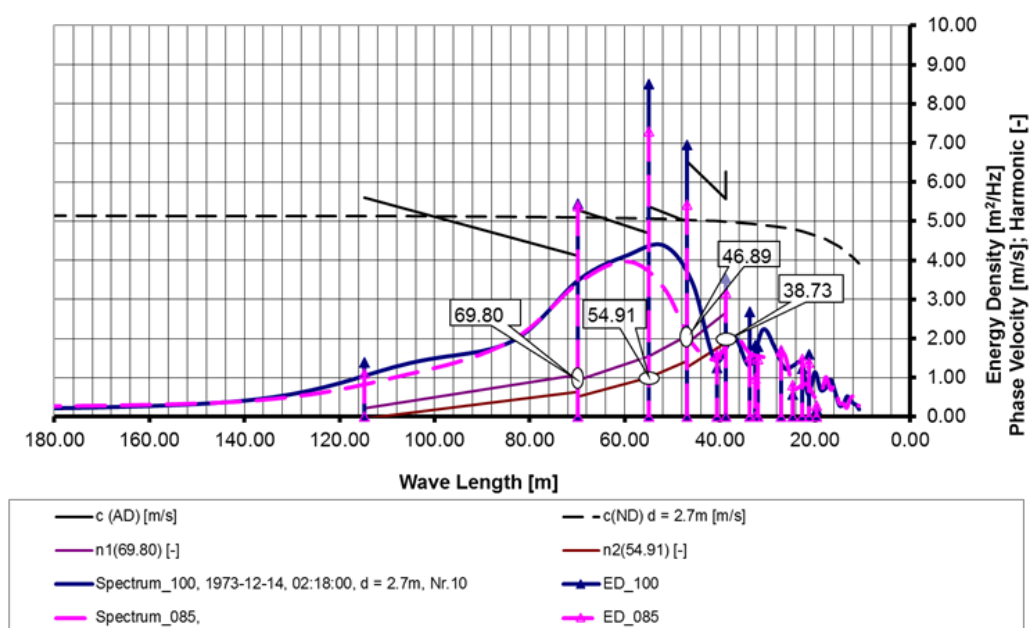


Fig.12: Energy density spectra transformed to the length axis in using the classical dispersion relationship. Line spectra of the energy density $ED(L)$, calculated on the basis of the anomalous Length Spectrum $L(AD)(f)$. In addition, the phase velocities and the ordinal numbers of the harmonic basin oscillations are plotted with reference to the wavelengths.

In Fig.10 the energy spectra of measurement No. 10 are shown together with the quasi measured (oscillatory) phase velocity function $c(AD)(f)$ and the Coherence-function. Additionally $c(ND)(f)$ is plotted according to the classical dispersion relation

The wave length spectrum $L(AD)(f)$ derived from $c(AD)(f)$ is to be seen in Fig.11 together with a smoothed (linearized) version $L(AV)(f)$ to be compared with the theoretical length spectrum $L(ND)(f)$ calculated for water depth $d = 2.7\text{m}$.

Moreover, the harmonic ordinal numbers of the two wave systems according to formula (1) basing on the first harmonics ($L_{1A} \approx 69.8\text{m}$ and $L_{1B} \approx 54.91\text{m}$ respectively) are shown. The second harmonics wavelength are $L_{2A} \approx 46.89\text{m}$ and $L_{2B} \approx 38.73\text{m}$.

Hence, the broad peak can be attributed notably to the presence of four resonance points belonging to the first and second harmonics of the two wave systems. This could yet have been read from the 4 oscillations in the phase velocity plot $c(AD)(f)$ of Fig.10 in the frequency range $0,05 \leq f \leq 0,135\text{Hz}$.

Using the generation law $L(n) = L(n=0)/(n+1)$ the lengths of the fundamentals can be estimated as $L_{0A} \approx 139.60\text{m}$ and $L_{0B} \approx 114.72\text{m}$. With respect to the distance beach – ridge, such values apparently are of some relevance, because energy density is still remarkable at those frequencies.

Fig.12 shows the transformation of the data from Figures 10 and 11 to the lengths – axis. Such a presentation seems to be useful, because the resonance points are marked not only by the ordinal numbers of the harmonics but also by jumps in the function of the phase velocities $c(AD)(L)$.

It is particularly noteworthy that several neighboring points of resonance, especially at high energetic storm surges, can form not only *multimodal* but also (wideband) *unimodal* energy density spectra.

The latter case can be identified by the joined existence of alternating sections of anomalous and normal dispersion in the frequency range possessing considerable energy densities.

Because of that feature, also the *global (averaged) anomalous dispersion*, considering nearly the whole energy containing frequency range, can be explained. By the way, this is similar to the total electromagnetic spectrum extending from radio waves to X-rays in case smoothing techniques are applied thereon; cf. Abb.2 in [18] or Fig.1 in [23] respectively.

A separate treatment on the formation of powerful *partial standing waves*, which in the end are necessary for maritime resonances to exist at the west coast of Sylt Island, is contained in [24]

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